

A method for bubble volume measurement under constant flow conditions in gas-liquid two-phase flow

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ABSTRACT

Measuring the volume of a bubble, especially at its detachment, is a basic subject in gas-liquid two-phase flow research. A new indirect method for this measurement under constant flow conditions is presented. An electronic device is designed and constructed based on laser beam intensity. This device calculates the frequency of the bubble formation by measuring the total time of the formation process and counting the number of bubbles crossing the laser beam. The bubble volume at detachment can be calculated by dividing the volumetric flow rate of air by the frequency of bubble formation. The latter and the bubble volume at detachment are measured for three different heights of water above the tip of the orifice (50, 100, and 150 mm), three orifice diameters (1, 2, and 3 mm), and different gas flow rates between 2000 and 10000 ml/hr. Comparing and validating the results with the results of the image processing (IP) method and the correlations presented by other studies shows the strong accuracy of the present method.

1. Introduction

The dynamics of bubble formation has great significance in diverse applications relating to the scattering of gas bubbles in liquids such as water purifying, chemical, and medical processes. This phenomenon is affected by different parameters such as the equivalent diameter of injection, geometry of injection, gas flow rate, the physical properties liquids and gases, height of the liquid column, and wettability. Under a constant flow condition, the frequency of the formation and the volume of the bubble have an inverse relationship with each other, and by knowing one of these

values, the other can be obtained by the means of the flow rate.

Certain tools that are usually used to measure the major parameters of bubble flow under different conditions are as follows:

- Thermocouples
- Optical probes
- Anemometer
- Electrical probes
- Photon attenuation techniques
- Image processing technique

1.1. Thermocouples

A classical thermocouple measures certain statistical characteristics of temperature and obtains data regarding the local void fraction in

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combination with an electrical phase indicator. As these measurements are based on the temperature difference between liquid and gas phases, this device is more appropriate for boiling two-phase flows. Wiebe and Judd [1] employed a 75 μm , chrome-constantan, micro-thermocouple in order to measure the superheat layer thickness in saturated and subcooled nucleate boiling. A time-averaged temperature was determined by integrating the temperature signal. In another research, Dhir et al. [2] used a rake of six thermocouples to measure the temperature in the thermal boundary layer. They used the data gathered from thermocouples to predict the effect of micro layer evaporation.

1.2. Optical probes

The optical probe is an instrument used for measuring local void fractions or interface passage frequencies. It is sensitive to change in the light refractive index of the surrounding medium. In this context, Davidson et al. [3] measured the frequency of bubble formation by stroboscopic illumination. According to experimental investigations, they concluded that the frequency of bubble formation decreases by increasing the orifice diameter. In another work, Abuaf et al. [4] utilized an optical probe with controlled-tip geometry. They found that the optical probe, with proper calibration, can be used to determine both the local void fractions and the interface velocities. Hamad et al. [5] used a dual optical probe for measuring the local volume fraction, drop velocity, and volume in the two-phase flow. The liquids were kerosene and water. The measurements of Hamad et al. showed the applicability of the probe. In another work, a four-point fiber-optical probe was used by Luther et al. [6] to estimate the bubble's aspect ratio and its velocity from the time series. They reconstructed the three dimensional shape of the bubble from a set of stereoscopic contour lines. Hooshyar et al. [7] used the four-point optical probe to investigate the velocity of rising bubbles in a liquid–solid suspension. They observed the effect of particles on the dynamics of a gas bubble. They presented a simple theory that describes the underlying mechanism in determining the terminal bubble velocity.

1.3. Anemometer

Although anemometers have been widely used for gases, it has been found that the hot-wire or

hot-film anemometry can be used in two-component two-phase flows. In this case, it is possible to measure the local void fraction and the turbulence intensity of the liquid phase in conjunction with the arrival frequency of bubbles or droplets. Goldschmidt and Eskinazi [8] measured the arrival frequency of liquid droplets 1.6–3.3 μm in diameter, with a constant temperature anemometer and a cylindrical probe of 4.5 μm in diameter. They inferred that when the impaction frequency of the droplets is different from the energetic frequency range of the turbulent gas stream, the signal fluctuations owing to the impacts could be distinguished from the fluctuations due to turbulence. Chuang and Goldschmidt [9] utilized the hot-wire as a bubble size sampler by theoretically investigating the character of the signal as a consequence of the travel of an air bubble past the sensor. Bremhorst and Gilmore [10] studied the response of hot wire anemometer probes into a stream of air bubbles in water flow. Three types of interactions were identified in their work: direct, glancing, and partial hits. Direct and glancing hits result in identical signal level changes and hence the discrimination between the bubble and the no-bubble signals; partial hits, on the other hand provide considerably reduced signal-level changes. Bhole et al. [11] utilized a laser Doppler anemometer to measure the flow pattern in a bubble column. A superficial gas velocity of 20 mm/s was used in all the experiments. From $z/D \geq 2$ (z : axial location; D : column diameter), fully developed axial liquid velocity profiles were seen. Furthermore, the radial variation of the mean axial velocity showed the gross liquid circulation in the column.

1.4. Electrical probe

Electrical probe is another instrument that can be used for observing bubble velocities, dimensions, and flow rates. Gunn and Al-doori [12] used this probe for measuring implicit properties. They found out that this probe has good accuracy when the conditions of electrical measurement were arranged to exclude spurious signals. Steinemann and Buchholz [13] presented an electrical conductivity technique for the measurement of bubble size and velocity. They found that the local interfacial area can be calculated by using the measurement of bubble sizes and the volumetric hold up of the dispersed phase. Saito et al. [14] applied neutron radiography and the electrical four-sensor conductivity probe to measure the

liquid–metal two-phase flow. They found that the measured radial void fraction profiles, which are obtained by neutron radiography and the electrical conductivity probe, agreed well with each other. Choi and Lee [15] compared four different probes, which included vertically projecting electroresistivity, horizontally projecting electroresistivity, U-shape light reflection, and light transmission. They found that the light transmission probe developed in their work could be used to effectively determine the bubble properties.

1.5. Photon attenuation techniques

Another instrument for measuring bubbles involved photon attenuation techniques. Gardner et al. [16] utilized the Gamma-ray-collimator for the measurement of the two-phase flow void fraction. Liu and Wang [17] used modified one-shot photon attenuation techniques for determining the average void fraction in the two-phase flow. They found that the accuracies of the void-fraction predictions were improved by this method. In addition, they also found that the void fractions predicted by the modified method were larger in comparison to those predicted by the conventional method.

1.6. Image processing technique

Most researchers utilized a high-speed camera and the image processing method to compute the frequency of bubble formation, which has high cost and is time consuming. For instance, Harvey et al. [18] measured vapor bubbles by using image analysis. They presented a method of computer image analysis, which determines the flow quantities of a single vapor bubble as it evolves near a rigid boundary. In another paper, Lguchi et al. [19] investigated the effects of cross-flow on the frequency of bubble formation from a single-hole nozzle by using a high-speed camera. They compared the frequency of bubble formation in a rotating bubble bath with respect to stationary bath and concluded that before the critical value of cross-flow velocity, the ratio of frequencies is unity, and after critical value of cross-flow velocity, the ratio changes in a complex manner. In another research, Badam et al. [20] perused the regimes of bubble formation by experimentally utilizing a high-speed camera. They found that at high gas flow rates, the bubble formation frequency remains constant and the bubble volume is almost independent of surface tension. In addition, Hanafizadeh et al. [21]

studied the formation, growth, and detachment of gas bubbles produced from a submerged needle in water by using a high-speed camera. They concluded that the bubble formation frequency is strongly dependent on the contact angle and the surface tension, and increases with the increase of the gas flow rate.

Although there are many techniques for measuring the characteristics of bubbles in the two-phase flow, most of these techniques involve high costs and need complex processing. Therefore, in the current work, a low-cost method for calculating the bubble volume indirectly at the detachments is presented. In this method, an electronic device (the Frequency Meter (FM)) has been designed and constructed based on the changes in the intensity of the light beam. By crossing bubbles through the beam, the intensity of the laser beam is changed. By counting numbers, the total time spent by the bubbles that pass through the beam and the frequency of formation are measured. Then, by considering the fact that the bubble volume multiplied by the frequency is equal to the volumetric flow rate, the bubble volume at detachment can be found. In addition, frequency and bubble volume is measured using the image processing (IP) method and it is compared with the results obtained by the FM device. Finally, the results of the FM device will be compared with popular correlations that are presented by other studies.

2. Experimental apparatus

The schematic of the experimental apparatus of the present work is shown in Fig. 1. The experimental system contains a compressor, a pressure regulator, a rotameter, a square column, orifices that are 1, 2, or 3 mm in diameter, a high-speed video camera (1200 fps), and a light source. For photographic observations, a square column was utilized, which was made from PMMA measuring dimensions 120 mm × 120 mm × 300 mm, and it was opened to the atmosphere at the top (Fig. 1-1). The heights of the liquid above the tip of the orifice are 50, 100, or 150 mm. In order to wipe out reflections, an 800 W halogen lamp was placed just in front of the camera (Fig. 1-6). The camera captures movies at 1,200 fps and 336 × 96 pixels (Fig. 1-7). The results are sent to a computer (Fig. 1-8) for image processing. The injection system contains the orifice plate located at bottom of the water column (Fig. 1-2) and a compressor, which is the source of the injection (Fig. 1-3). A pressure

regulator controls the pressure of the injection line (Fig. 1-4). A rotameter with a high precision needle valve is placed just before the orifice in order to measure and adjust the flow rate (Fig. 1-5). The flow rate range is between 2,000 and 10,000 ml/hr. For measuring the frequency of formation, a laser device with a power rating of 200 mW has been utilized (Fig. 1-11). The laser device is placed 1 cm above the tip of the orifice to ensure that all of the bubbles pass through the laser beam. The receiver sensor and the FM device are placed just in front of the laser to identify the intensity of the laser beam (Figs. 1-9 and 1-10).

The range of the operating conditions can be seen in Table 1 (the physical properties are computed at 20°C).

When the parameters are equal to the experimental measurement values, certain uncertainties, owing to measurement limitations, are present. The uncertainties in the measured data are shown in Table 2. In this table, the precision of the major parameters is reported based on the accuracy of the measuring instruments. Moreover, the uncertainty of bubble diameter and volume are computed according to the analysis of uncertainty of the Lazar study [22].

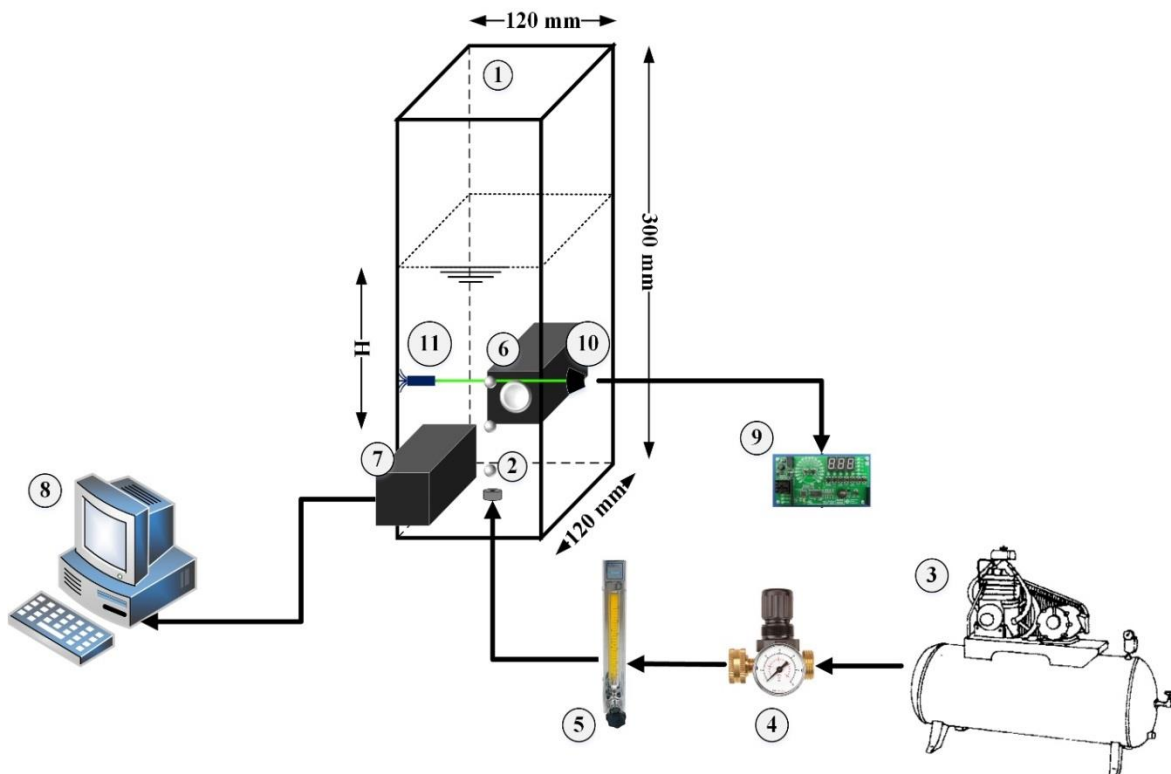


Fig. 1. Schematic diagram of experimental apparatus: (1) bubble column, (2) orifice plate, (3) compressor, (4) pressure regulator, (5) rotameter, (6) light source, (7) high-speed camera, (8) computer, (9) electronic board, (10) sensor, and (11) 200 mW laser

Table 1. Range of operating conditions.

Operating Parameters	Parameter Value
Flow rate	2,000, 4,000, 6,000, 8,000, and 10,000 mlph
Orifice diameter	1, 2, and 3 mm
Liquid height above the needle	50, 100, and 150 mm
Liquid viscosity	0.001 Pa. s
Surface tension	$72.5 \times 10^{-3} \text{ N. m}^{-1}$
Water Density	$997.05 \text{ Kg. m}^{-3}$
Air Density	1.15 Kg. m^{-3}

Table 2. Uncertainty of the experimental measured variables.

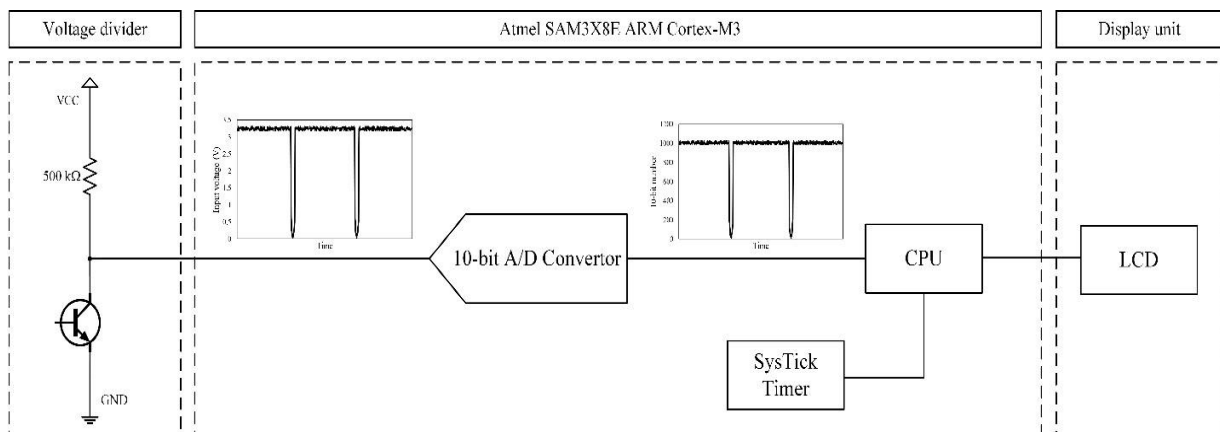
Parameters	Amount of Uncertainty	Percentage Uncertainty
Air flow rate	–	5%
Orifice diameter	0.005 mm	–
Physical calibration scale	0.005 mm	–
Image calibration scale	1 pixel	–
Bubble diameter	0.05 mm	–
Bubble volume	0.1 mm ³	–

3. Methodology

As mentioned before, this method is based on the refraction of light caused by the bubbles crossing the light beam. An electronic circuit is designed to count the number of bubbles and compute the time that is spent in each bubble crossing the light beam. An Atmel SAM3X8E ARM Cortex-M3 microcontroller is chosen due to its 84 MHz clock speed and the 32-bit architecture ARM core. Figure 2 illustrates the operation diagram of the circuit. A phototransistor is used as a sensor for measuring the received light intensity as shown in Fig. 1. It is an analog sensor in which the electrical resistance changes with the received light intensity. A voltage divider is used for converting the electrical resistance of the sensor to variable voltage at the center tap. Then, this voltage is read by a 10-bit internal analog to digital convertor of the microcontroller. The SysTick timer is a 24-bit counter, which is one of the features of this microcontroller that adjusted to count every millisecond; therefore, time could be measured in milliseconds. Then, the CPU that receives the data computes the

frequency of bubble formation and the average volume of the bubbles, and displays it on a graphical LCD.

The flowchart of the computing frequency of the formation and volume of the bubble is presented in Fig. 3. As shown in this figure, if the start button is turned “on”, the microcontroller measures the sensor value (the 10-bit number assigned to the voltage of the voltage divider in the range of 0–3.3 V) continuously and records this value with its measured time to the serial port for analysis. During measurement, if the sensor value becomes lower than a specified value named “MinValue”, it means that a bubble has interrupted the laser beam. If this happens, the time will be saved in an array and the number of bubbles (i) will be increased by one. Then, a loop will run until the sensor value increases up to MinValue, which means that the bubble has completely crossed the light beam. These steps will be repeated while the start button is turned “on”. After the start button is turned “off”, the bubble frequency and the average volume of the bubbles will be calculated.

**Fig. 2.** Schematic of the operation diagram of the circuit of the present paper.

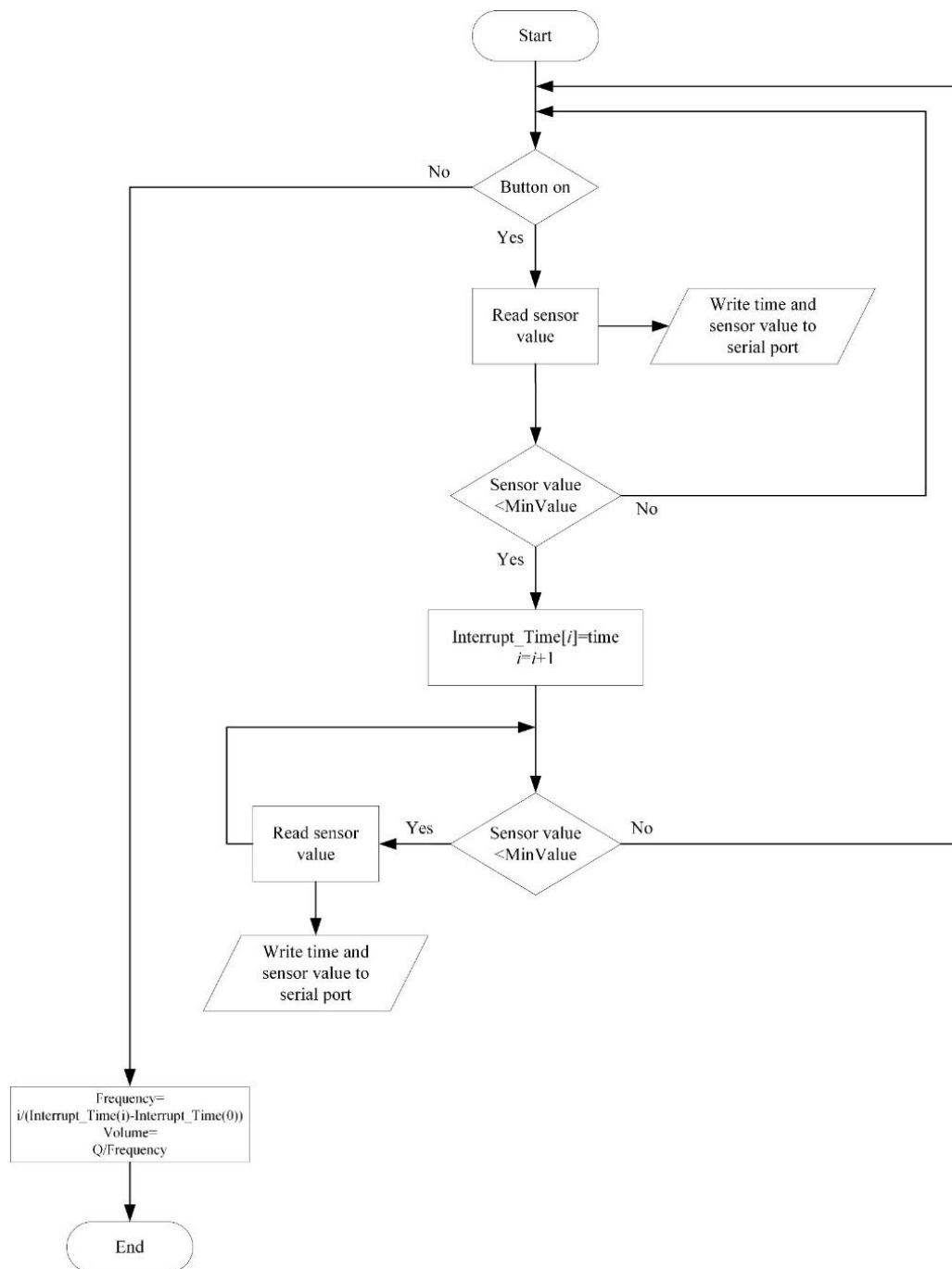


Fig. 3. The algorithm of computing frequency and volume of the bubble.

4. Results and Discussion

The charts of the sensor's response to the laser beam are shown in Fig. 4 for the volume rates of 2,000 and 4,000 ml/hr, with liquid height of 5 cm and orifice diameter of 2 mm. The y and x axes track the input voltage and the time, respectively. As the bubbles cross the light beam, the voltage falls to a near-zero value.

Therefore, by considering the number of dips in the graph and the duration of time in them, the frequency of bubble formation can be computed. On the other hand, as seen in the figure, by increasing the flow rate from 2,000 ml/hr (Fig. 4-a) to 4,000 ml/hr (Fig. 4-b), the dips of the chart get closer, which means that the frequency of formation increases.

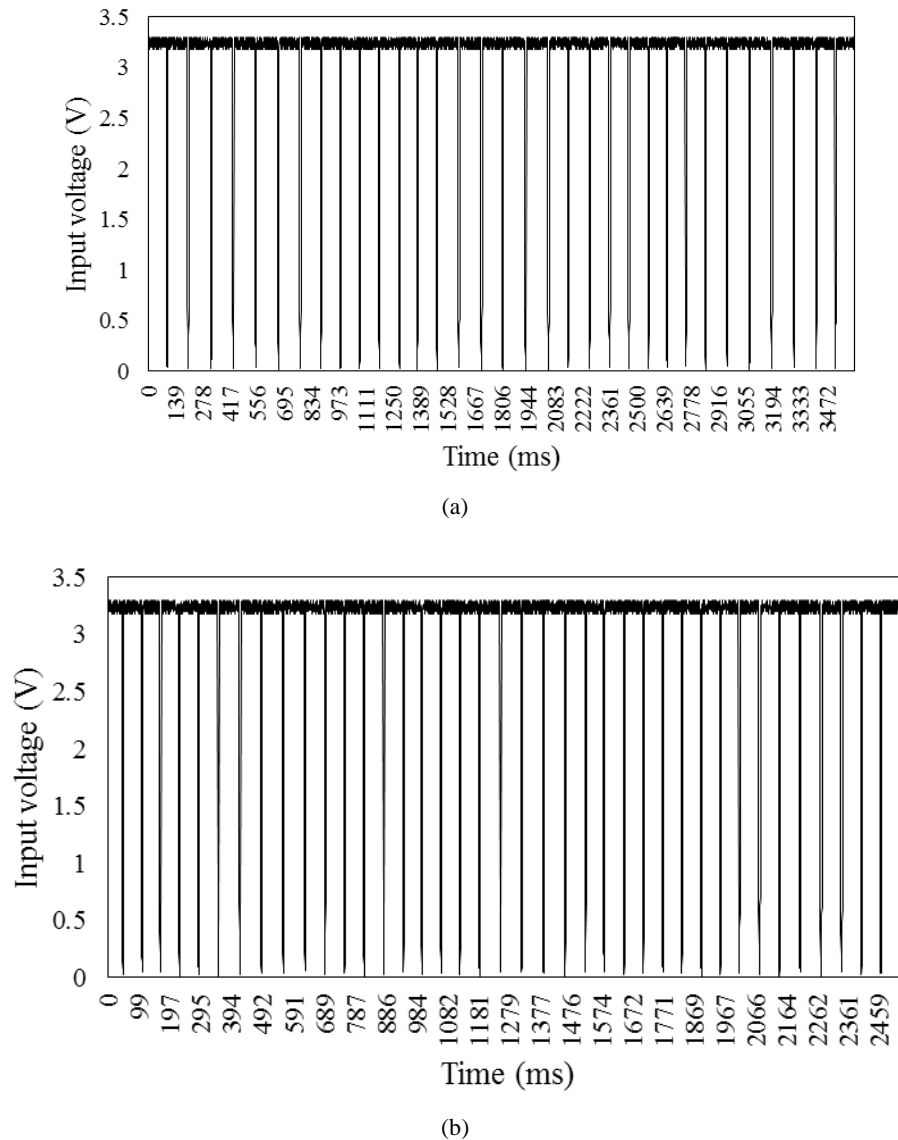


Fig. 4. Fluctuation time for flow rates of (a) 2,000 and (b) 4,000 (in ml/hr) gathered by FM device for liquid height of 5 cm and orifice diameter of 2 mm.

Figure 5 shows the frequency of formation obtained from the FM device versus the values obtained from the IP technique for different orifice diameters and a liquid height of 5 cm above the needle. As shown in the figure, the data of the FM device are in good agreement with the IP method. Based on the analysis for all tests performed in this research, the results of the FM device have a maximum error of ± 2 percent with respect to the IP method. This means that the results are accurate and reliable.

Figure 6 shows the frequency of bubble formation measured by the FM device versus the flow rate for different orifice diameters and a liquid height of 5 cm above the orifice. By

increasing the dispersed phase flow rate, the upward forces acting on the bubbles become larger than the forces acting downward. Thus, the bubbles detach faster from the orifice and this results in higher frequencies of formation. On the other hand, it is obvious that the bubble diameters for the 2-mm orifice are smaller than the diameters for 3- and 4-mm orifices. As a result, the bubble frequency decreases with increase in the orifice diameter. As a bubble leaves the orifice, a small amount of liquid seeps into the air chamber placed below the orifice. This phenomenon is termed “weeping” and has been addressed by a number of researchers [24–26].

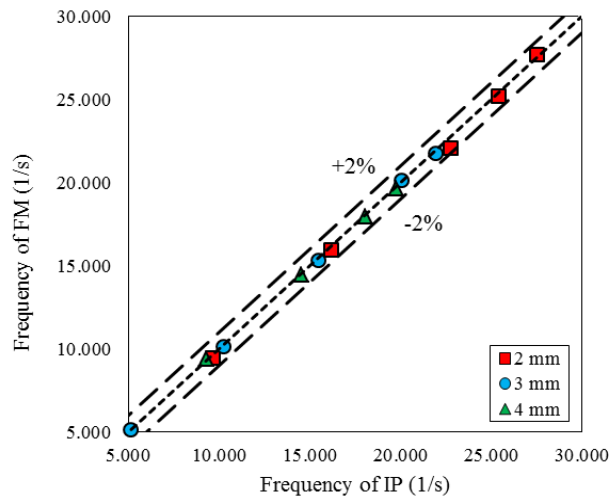


Fig. 5. Frequency of bubble formation obtained from the FM device versus the frequency of the formation measured by the IP technique for a liquid height of 5 cm above the tip of the orifice.

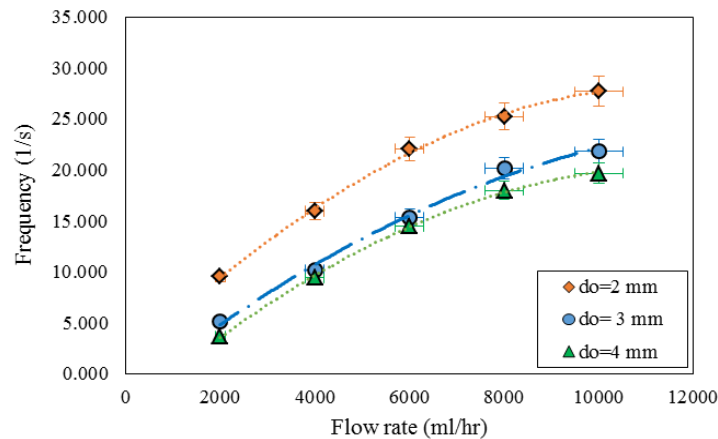


Fig. 6. Frequency of the bubble formation obtained from the FM device versus the flow rate for a liquid height of 5 cm above the tip of the orifice.

The dependency of the frequency of formation (measured by the FM device) on the liquid height for different liquid heights and the 2-mm orifice diameter is shown in Fig. 7. It is obvious from the figure that the liquid height does not have a significant effect on the frequency of formation. This is in agreement with the results of Jamialahmadi et al. [27].

In the constant flow condition, the volume of the bubble at detachment and the frequency of formation have the following relationship:

$$f = \frac{Q}{V} \quad (1)$$

in which f , Q , and V are the frequency of formation, volumetric flow rate, and volume of the bubble at detachment, respectively. Thus, by knowing the flow rate and the frequency of

formation, the bubble volume can be easily computed.

By measuring the frequency of formation by the FM device, the volume of the bubbles in the different flow rates, orifice diameter, and liquid height have been computed. To put the accuracy of this method into perspective, the most important correlations are presented for calculating the volume of the bubble at detachment, which are compared with the experimental data in Fig. 8. The correlation presented by Dietrich et al. [28] covers relatively low flow rates. Hence, by increasing the gas flow rate, the error of the correlation increases. The best match correlation for the experimental data of the present work is the correlation of Jamialahmadi et al. [27], which demonstrated the greatest error of 10 percent with the experimental data of the present

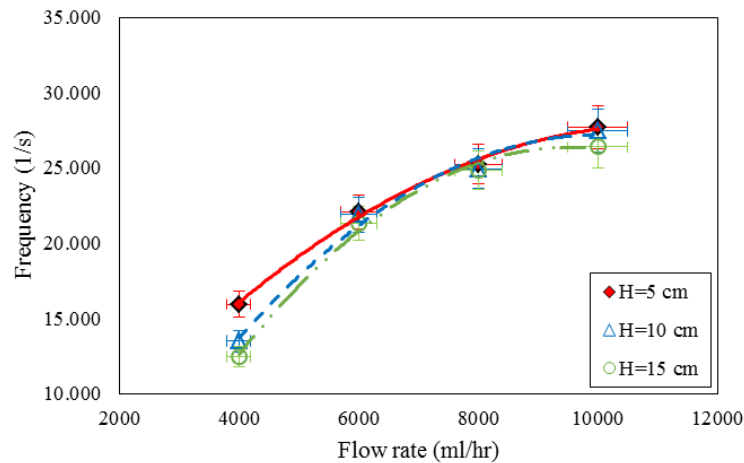


Fig. 7. Frequency of the bubble formation obtained from the FM device versus the flow rate for liquid height of 5 cm above the tip of the orifice.

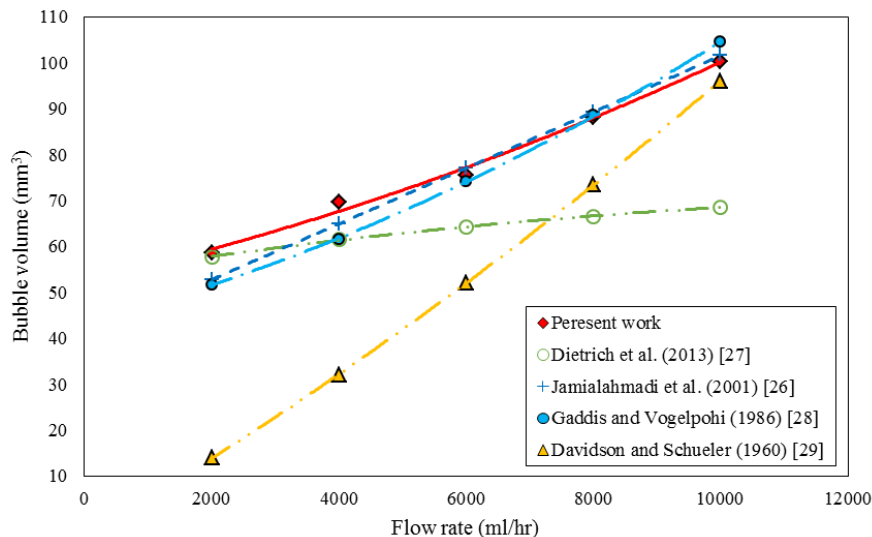


Fig. 8. Prediction of the bubble diameter from various correlations and the experimental data of present work for an orifice diameter of 2 mm and a liquid height of 5 cm.

research. Furthermore, the model presented by Gaddis and Vogelpohl [29] shows good agreement with the experimental data. Davidson and Schüler's model [30] has the least accuracy in low flow rates among the other correlations, although by increasing the flow rate, prediction of this correlation get closer to the experimental data.

5. Conclusion

An electronic device is designed and constructed based on the change in the laser beam intensity for measuring the frequency of formation and the bubble volume at detachment in the constant flow condition. Comparing the bubble volume at

detachment, which is calculated by this device for different heights and orifice diameters with the IP method, shows an error lower than 2 percent. It is found that with an increase in the gas flow rate, the frequency of formation increases. Furthermore, the bubbles generated from the lower orifice diameters have a higher frequency of formation. On the other hand, it is found that the liquid height above the orifice does not have a significant impact on bubble formation. Finally, in comparison to the results of the correlations presented by other researchers, it is concluded that the best-match correlation to the present data has been suggested by Jamialahmadi et al. [27].

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